

AEB 6933 Mathematical Statistics—Examination II
November 13, 2006

- I. Assuming that the data in Column 2 of Table 1 are distributed Pareto

$$f(x; k, x_m) = k \frac{x_m^k}{x^{k+1}}, \quad x \geq x_m$$

with $x_m = 5$:

- a. Estimate the shape parameter k using maximum likelihood. (15 points)
 - b. What is the variance of the estimate of k and why? (15 points)
 - c. Test $H_0: k \geq 0.75$ against $H_1: k < 0.75$. (15 points)
- II. Using the data in Column 5 of Table 1 and assuming $y \sim N(\mu, \sigma^2)$, test for $H_0: \mu = 6.0$ against $H_1: \mu \neq 6.0$. How does the test change if the variance is known and equal to 2.5? (15 points)
- III. Suppose we start with a prior distribution $g(p) = -p(p-1)$ for the Bernoulli distribution. The posterior distribution then becomes

$$\begin{aligned} f(p|x) &= \frac{p^x (1-p)^{1-x} [-p(p-1)]}{\int_0^1 p^x (1-p)^{1-x} [-p(p-1)] dp} \\ &= \frac{24 (1-p)^{1-x} (1-p) p^{1+x}}{\Gamma(3-x) \Gamma(2+x)} \end{aligned}$$

Implying

$$f(p|x) = \begin{cases} 12 (p-1)^2 p & \text{for } x=0 \\ 12 (p-1) p^2 & \text{for } x=1 \end{cases}$$

Compute the expected value of p for each possible outcome of x . What is the probability of $p \leq 0.5$ if $x=1$? (20 points)

- IV. When are ordinary least squares estimates also the maximum likelihood estimators? (20 points)

Table 1. Empirical Data

Observation	x	$\ln x$	x^2	y	y^2
(1)	(2)	(3)	(4)	(5)	(6)
1	10.015	2.304	100.291	3.523	12.415
2	46.011	3.829	2116.968	3.722	13.852
3	5.760	1.751	33.174	7.036	49.504
4	97.633	4.581	9532.111	6.384	40.749
5	605.676	6.406	366843.391	3.721	13.849
6	17.319	2.852	299.964	2.845	8.094
7	8.300	2.116	68.894	3.192	10.192
8	37.797	3.632	1428.610	6.620	43.828
9	5.805	1.759	33.698	6.292	39.589
10	48.033	3.872	2307.185	0.759	0.576
11	8.091	2.091	65.465	2.003	4.013
12	386.114	5.956	149084.104	5.231	27.358
13	5.900	1.775	34.815	5.570	31.026
14	5.502	1.705	30.268	3.560	12.677
15	40.093	3.691	1607.455	2.047	4.191
16	5.516	1.708	30.425	7.243	52.463
17	22.296	3.104	497.091	4.833	23.358
18	7.595	2.027	57.682	4.219	17.796
19	61.843	4.125	3824.553	7.545	56.921
20	5.725	1.745	32.781	5.915	34.989
Sum	1431.023	61.029	538028.923	92.260	497.438

Table 2. Probability Levels for the Chi-Squared Distribution

Degrees of Freedom	Probability Level					
	0.99	0.95	0.90	0.10	0.05	0.01
1	0.000	0.004	0.016	2.706	3.841	6.635
2	0.020	0.103	0.211	4.605	5.991	9.210
3	0.115	0.352	0.584	6.251	7.815	11.345
4	0.297	0.711	1.064	7.779	9.488	13.277
5	0.554	1.145	1.610	9.236	11.070	15.086
6	0.872	1.635	2.204	10.645	12.592	16.812
7	1.239	2.167	2.833	12.017	14.067	18.475
8	1.646	2.733	3.490	13.362	15.507	20.090
9	2.088	3.325	4.168	14.684	16.919	21.666
10	2.558	3.940	4.865	15.987	18.307	23.209

Table 3. Probability Levels for
Student's t and Normal
Distributions

Degrees of Freedom	Probability Level		
	0.90	0.95	0.99
1	3.078	6.314	31.821
2	1.886	2.920	6.965
3	1.638	2.353	4.541
4	1.533	2.132	3.747
5	1.476	2.015	3.365
10	1.372	1.812	2.764
15	1.341	1.753	2.602
20	1.325	1.725	2.528
25	1.316	1.708	2.485
30	1.310	1.697	2.457
40	1.303	1.684	2.423
Infinity	1.282	1.645	2.326